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1. The amount of radioactive material, Q grams, decays according to the model given by the equation $Q = 200 \times 10^{-kt}$, where t is measured in years. It is known that after 40 years, the amount of radioactive material present is 50 grams.

a) find the value of k.

$50 = 200(10)^{-k(40)}$

$\frac{1}{4} = 10^{-40}$

$\frac{\log(\frac{1}{4})}{-40} = \frac{-40k}{-40}$

$k \approx 0.0151$ $\boxed{STO}\to x$

b) Find the amount of radioactive material present after 80 years.

$Q = 200(10)^{-80x}$

$Q = 12.5$ grams

c) What is the half-life for this substance?

$100 = 200(10)^{-xt}$

$\frac{1}{2} = 10^{-xt}$

$\frac{\log(\frac{1}{2})}{-x} = \frac{-xt}{-x}$

$t = 20$ years

2. The population number N in a small town in northern India is approximately modeled by the equation $N = N_0 \times 10^{kt}$, $t > 0$, where N_0 is the initial population and t is the time in years since 1980. The population was found to increase from 100,000 in 1980 to 150,000 in 1990. $t = 10$

$t = 0$ in 1980

$N_0 = 100{,}000$

a) Find the value of k.

$150{,}000 = 100{,}000(10)^{k(10)}$

$1.5 = 10^{10k}$

$\frac{\log(1.5)}{10} = \frac{10k}{10}$

$k \approx 0.0176$ $\boxed{STO}\to x$

b) Find the population in this town in 1997. $t = 17$

$N = 100{,}000(10)^{17x}$

$N \approx 199230$

c) How long will it be before the population reaches 250,000?

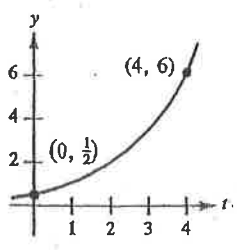
$250{,}000 = 100{,}000(10)^{xt}$

$2.5 = 10^{xt}$

$\frac{\log 2.5}{x} = \frac{xt}{x}$

$t \approx 22.6$ years.

3. Find the constant k such that the exponential function $y = Ce^{kt}$ passes through the given points on the graph.



for $(0, \frac{1}{2})$ $\frac{1}{2} = Ce^{k(0)}$

$C = \frac{1}{2}$ $(e^0 = 1)$

for $(4, 6)$ $6 = \frac{1}{2}e^{k(4)}$

$12 = e^{4k}$

$\frac{\ln 12}{4} = \frac{4k}{4}$

$k \approx 0.6212$

4. If $\log_a b = 5$ and $\log_a c = 0.2$

find $\log_a\left(\frac{\sqrt{b}}{ac^3}\right)$

$\frac{1}{2}\log_a b - \log_a a - 3\log_a c$

$\frac{1}{2}(5) - 1 - 3(0.2)$

$1.5 - 0.6$